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NATURAL FREQUENCY MEASUREMENTS FOR ROTATING SPANWISE UNIFORM CANTILEVER BEAMS

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1. INTRODUCTION

An Euler–Bernoulli cantilever beam attached to a rotating body often serves as the representation of mechanical systems such as helicopter or turbine blades, robotic arms, and satellite appendages. A well-known result from a free vibration analysis is that as the (constant) rotation speed increases, the natural frequencies of the beam increase as well [1]. In fact, this "stiffening" effect has been measured on rotor blades in the helicopter and engine industries.

The purpose of this letter is to present experimental natural frequency measurements for the geometrically simple case of a spanwise uniform, cantilever beam in the plane of rotation (lead/lag motion) and out of the plane of rotation (flapping motion) for various (constant) rotation speeds. This work demonstrates, as is theoretically predicted, that centrifugal stiffening terms in simple rotating beam models cannot be neglected even at first order. Perhaps, this will help to put to rest claims to the contrary which occasionally find their way into publication; see, for example, the work discussed in reference [2].

2. THEORETICAL BACKGROUND

An analytical model is presented to serve as a basis for the experimental modal survey. Figure 1 illustrates a flexible cantilever beam attached to a rigid body which rotates with a constant angular speed, Ω, about a fixed axis in inertial space. The rigid body has a radius R. The beam is uniform with length, L; cross-sectional area, $A = (a \times b$ for the rectangular cross-section shown); density, ρ; moments of inertia, I_2 and I_3 about the $\mathbf{a}_2$ (lead/lag) and $\mathbf{a}_3$ (flapping) directions, respectively, and Young's modulus, E. $V(W)$ denotes displacement in the lead/lag (flapping) direction.

To first order, the linear equations of motion in the lead/lag and flapping directions lead to the eigenvalue problems [3]

$$(A + v_3^2 B - I)[V] = \lambda_v [V] \quad \text{and} \quad (A + v_2^2 B)[W] = \lambda_w [W], \tag{1}$$

where

$$v_i^2 = \frac{EI_i}{\rho A l^4 \Omega^2}, \quad i = 2, 3. \tag{2}$$

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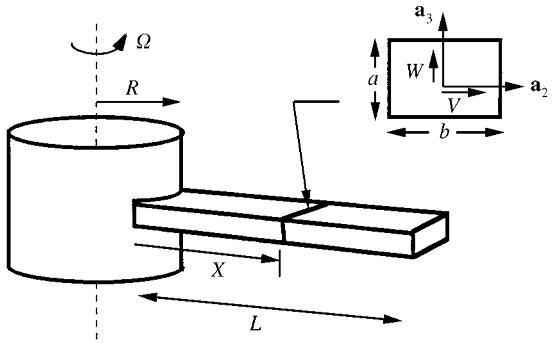

Figure 1. Flexible cantilever beam of length, L, and uniform cross-section, $A = ab$, is attached to a rigid body of radius R which rotates with angular speed Ω. Displacement co-ordinates (V, W) are defined with respect to the unit vectors $\mathbf{a}_2$ and $\mathbf{a}_3$. The beam is uniform, with Young's modulus E and mass density ρ.

The operators A, B, and I are defined as

$$A = (x + R)(\cdot)_{,x} + \tfrac{1}{2}\left[x^2 - 1 + 2R(x - 1)\right](\cdot)_{,xx},$$

$$B = (\cdot)_{,xxxx}, \quad \text{and} \quad I = (\cdot).$$

Eigensolutions to equation (1) yield the natural frequencies, ω_v and ω_w, from $\lambda_v = \omega_v^2/\Omega^2$ and $\lambda_w = \omega_w^2/\Omega^2$. The frequencies increase in magnitude as the rotation speed, Ω, increases (with all other parameters held constant). The numerical results presented herein are based on the Galerkin's method using non-rotating cantilever bending modes as basis functions.

3. EXPERIMENTAL APPARATUS

A schematic diagram of the experimental apparatus is shown in Figure 2 and consists of the following: (a) vacuum vessel–aluminum tank which is evacuated to eliminate

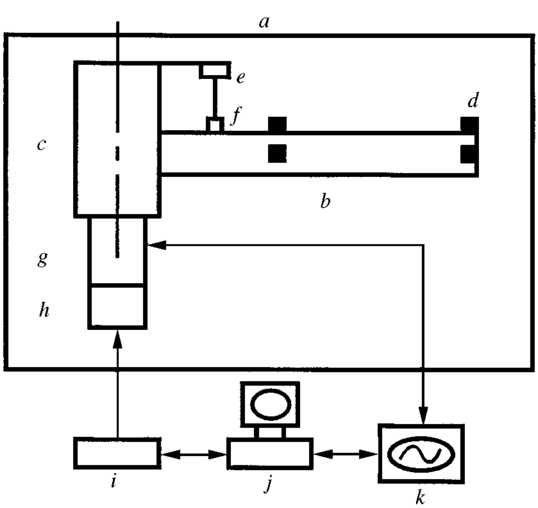

Figure 2. Schematic of experimental apparatus: (a) vacuum vessel; (b) flexible beam; (c) rigid body; (d) accelerometers; (e) vibration exciter; (f) load cell; (g) slip ring; (h) electric motor; (i) motor controller, (j) data acquisition computer; (k) signal generator/analyzer.

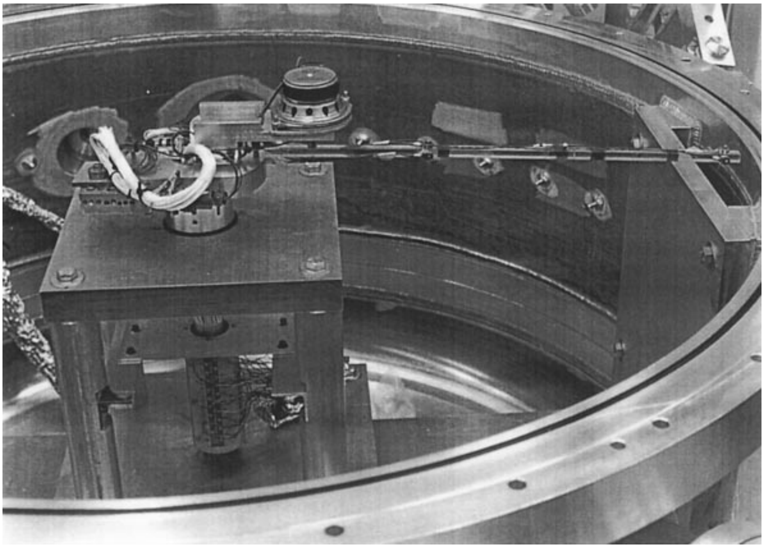

Figure 3. Photograph of hub/beam assembly including vibration exciter and accelerometers in open vacuum tank.

aerodynamic loads; (b) flexible beam–Lexan, length = 0·52 m, Young's modulus = 2375 MPa, density = 1200 kg/m^3; (c) rigid body–solid, aluminum hub, radius R = 0·076 m, mounted to a steel drive shaft (the beam extends over the diameter of the rigid body and is bolted down at two locations to better create a fixed boundary condition); (d) eight accelerometers–Endevco 7250 A/AM-10, mounted at four locations along the length of the beam with each pair mounted normal to the motion in the flapping or lead/lag directions; (e) vibration exciter–generic audio speaker mounted at an angle to excite motion in both the flapping and lead/lag directions; (f) load cell–Wilcoxon Research 5969, mounted on the beam and connected to the exciter by a short, stiff stinger; (g) slip ring-36 channels, allows wire free connection for signals from the accelerometers, exciter, and load cell; (h) electric motor–Parker Hannifin OEM650 Microstep Drive with OEM83-135-MO Step Motor run at 25 000 steps/rev; (i) motor controller–Galil DMC-1010 Motion Controller Card in personal computer; (j) data acquisition and analysis computer running SMS STAR, a modal analysis software package that determines the natural frequencies, shapes, and damping ratios of vibration modes within an excitation frequency range; (k) signal generator/analyzer-HP3566A. A photograph of the apparatus is shown in Figure 3.

4. MEASUREMENT PROCEDURE

Results from the modal survey are presented as four cases. In each case, the beam cross-section is taken to be square ($a = b$) with normalized dimensions (with respect to the shortest $a = b$ values of the test beams = $1{\cdot}83 \times 10^{-2}$ m): Case 1: $1{\cdot}00 \times 1{\cdot}00$; Case 2: $1{\cdot}17 \times 1{\cdot}17$; Case 3: $1{\cdot}67 \times 1{\cdot}67$; and Case 4: $2{\cdot}00 \times 2{\cdot}00$. It is important to note that in actuality the modal survey was conducted on eight beams with rectangular cross-section dimensions in combinations of the above dimensions so that the lead/lag and flapping natural frequencies were well separated and thus easier to differentiate at low rotation speeds.

The natural frequencies of the beams were determined from accelerometer response measurements to random excitation applied by the vibration exciter. For reference, the

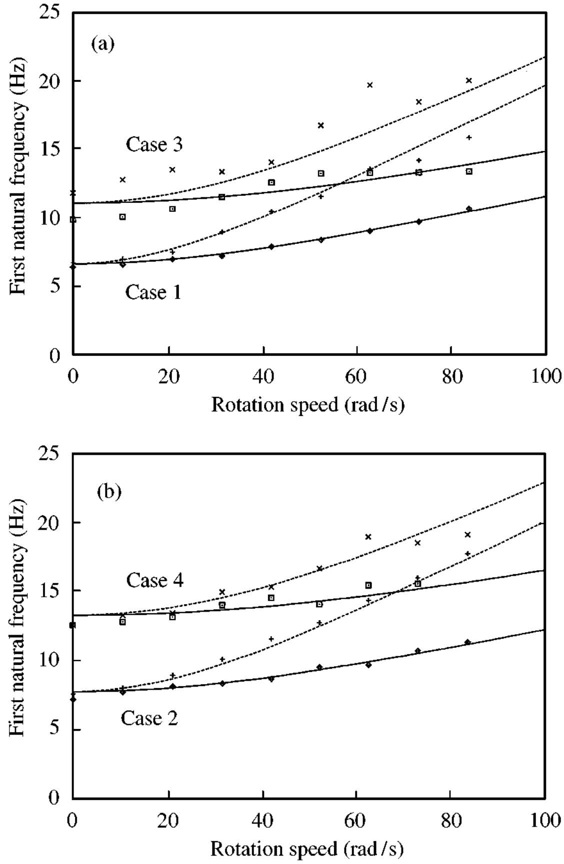

Figure 4. Comparison of theoretical and experimental first natural frequencies (Hz) for various rotation speeds (rad/s). Theoretical results: (——), lead/lag; (- - - -), flapping. Experimental results: (◇, □), lead/lag; (+, ×), flapping; (a) Case 1 ($a = b = 1.00$) and Case 3 ($a = b = 1.67$); (b) Case 2 ($a = b = 1{\cdot}17$) and Case 4 ($a = b = 2.00$).

response of each beam was measured for the stationary case ($\Omega = 0$). Using Euler–Bernoulli cantilever beam theory, the non-dimensional bending stiffness, $\sqrt{EI/\rho Al^4}$, was determined for each beam in each direction, i.e., from $\sqrt{EI/\rho Al^4} = \beta_n^2/\omega_{n\,measured}$, where β_n (n is the mode number) is known for stationary cantilever beams. The average value of the non-dimensional bending stiffness was calculated for all of the beams in the survey.

To begin a test, the vacuum tank was evacuated and the motor was run up to a specified, constant rotations speed. After a short period of time, transients died out and the beam reached a steady state. Random excitation was applied over a 17·5 Hz frequency range. The signal analyzer recorded 6–10 averages of 800 lines of data per average over the excitation frequency range. The excitation amplitude was adjusted to insure sufficient output response, greater than the noise level. This procedure was repeated for eight discrete rotation speeds. The resulting acceleration data was analyzed using the STAR software.

The rotating imbalance of the hub/beam was minimized by careful placement of counter weights on the hub. However, the imbalance could not be completely eliminated. The residual imbalance excited natural frequencies of the hub/motor assembly and the vacuum tank. Measurable harmonics were noted at multiples of 6 Hz and the vacuum tank had a significant mode at approximately 400 Hz. These frequencies were easily identified by the modal analysis software. There was a significant noise contamination of the acceleration response measurements at higher rotation speeds for the wider/thicker beams.

5. EXPERIMENTAL RESULTS

The first bending natural frequencies in the lead/lag and flapping directions for the four cases are shown in Figure 4(a) and 4(b). They are plotted with frequencies calculated from equation (1). The theoretical predictions used an average value for the non-dimensional stiffness which accounts for the differences between the theoretical and measured frequencies at $\Omega = 0$.

The experimentally determined natural frequencies (diamonds or squares for the lead/lag direction, pluses or x's for the flapping direction) compare closely to the theoretical predictions (solid line for lead/lag, dashed line for flapping). The experimental results clearly show the effect of the centrifugal stiffening in both directions. Correlation is best for the beams with smaller cross-sections. The scatter increases with cross-section size. With the larger beams, the rotating imbalance was greater causing contamination of the response signal. In fact, the first natural frequency in the lead/lag direction for Case 4 at the highest rotation speed could not be determined from the response signal due to a poor signal-to-noise ratio. It was noted that the signal-to-noise ratio decreased as the rotation speed or cross-section dimension increased. The vibration exciter was not able to generate measurable motion for stiffer beams. Note that for rectangular cross-sections for which $a < b$, the natural frequencies in the two directions cross-over. For example, for Cases 2 and 4 the flapping natural frequency crosses over the lead/lag natural frequency near $\Omega = 65$ rad/s. Under finite strain, this may lead to internally resonant response.

Similar results for the second natural frequencies were obtained but are not presented here. The separation between lead/lag and flapping natural frequencies is smaller than that for the first natural frequencies making identification more difficult.

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